**Are there mathematical hinges?[[1]](#footnote-1)\***

**Abstract**: In this paper I argue that, contrary to what several prominent scholars of *On Certainty* have claimed, Wittgenstein did not maintain that simple mathematical propositions like “2x2=4” or “12x12=144”, much like G. E. Moore’s truisms, could be examples of hinge propositions. In particular, given his overall conception of mathematics, it was impossible for him to single out these more simple mathematical propositions from the rest of mathematical statements, to reserve only to them a normative function. I then maintain that these mathematical examples were introduced merely as objects of comparison to bring out some peculiar features of the only hinges he countenanced in *On Certainty*, which were all outside the realm of mathematics. I then close by gesturing at how the distinction between mathematical hinges and non-hinges could be exemplified and by exploring its consequences with respect to (Wittgenstein’s) philosophy of mathematics.

1. *Introduction*

In *On Certainty* there are several remarks in which Wittgenstein discusses simple mathematical propositions like “2x2=4” or “12x12=144”.[[2]](#footnote-2) And one may wonder whether he thought that these mathematical trivialities, much like G. E. Moore’s truisms in “A defence of common sense” (1925), could be examples of hinge propositions. If they were, they would have to be importantly different from other mathematical propositions, because of the peculiar role they would play. To get a sense of the difference, they would have to play a role like the one of “The Earth has existed for a long time” with respect to ordinary propositions about the past such as “There were dinosaurs on the Earth over 230 million years ago”. While the latter proposition is open to verification and control and may turn out to be true or false, the former is not open to such verification and control and, for Wittgenstein, it is neither true nor false, or, if it is true, it is so in a totally “empty” sense (McGinn 1989: 128). Indeed, according to Wittgenstein, hinges play a rule-like role (OC 95), while ordinary empirical propositions do not. If there were mathematical hinges, they would have to play such a normative function, while other mathematical propositions should not be akin to rules.

The trouble, however, is that for Wittgenstein (at least on the *vulgata*) *all* mathematical propositions are rules and allegedly false ones, like “2+2=5”, are in fact *meaningless*, *not false*. Thus, in the realm of mathematics, as Wittgenstein thought of it,[[3]](#footnote-3) it seems we don’t have a contrasting class with respect to which we could say that only some mathematical propositions would have a hinge status. Of course, one might reject Wittgenstein’s own conception of mathematical propositions as rules, to maintain the more traditional view that they are genuine propositions, subject to truth and falsity. Or else, one may dispute the *vulgata* and offer a different interpretation of Wittgenstein’s views on mathematics. In that case, however, one would still be left with the task of explaining why only some mathematical propositions (or statements),[[4]](#footnote-4) like “2x2=4” and “12x12=144”, would be rules, which are neither true nor false, or, if they are true, they are so in an entirely “empty” sense. Thus, one would be left with the problem of explaining what makes such a subclass of mathematical propositions special.

We then seem to be caught in a dilemma. For, on the *vulgata* concerning Wittgenstein’s views of mathematics, there is no (clear) distinction between “normal”, or “ordinary” mathematical propositions, on the one hand, and mathematical “hinges”, on the other. In contrast, on a different understanding of mathematical propositions, the distinction could be drawn conceptually, for we could conceptually differentiate between “ordinary” and “hinge” mathematical propositions, but it would be difficult to maintain that the examples provided by Wittgenstein in *On Certainty* could be used to exemplify it.

In the rest of this paper, I would like to explore the issue further to see whether, after all, sense can be made of the idea of mathematical hinges. The upshot, on closer inspection, is that, contrary to what several prominent scholars of *On Certainty* have maintained,[[5]](#footnote-5) Wittgenstein did not really think of mathematical truisms such as “2x2=4” and “12x12=144” as hinges. Rather, he introduced them as objects of comparison, to clarify or stress some properties of the only hinges he really countenanced, which were all outside the domain of math.

Towards the end, however, I will also gesture at how, in my view, the idea of mathematical hinges could be maintained. This, in turn, will have important consequences with respect to an appraisal of Wittgenstein’s ideas about mathematics. Furthermore, if correct, it will give us the basics for a possible “hinge philosophy of mathematics”, whose development will have to be left for another occasion.

1. *Hinges*

Apart from examples, and *On Certainty* contains plenty of them, we can individuate a few criteria that distinguish hinges from ordinary empirical propositions, for Wittgenstein. The set of criteria is itself an object of debate among Wittgenstein scholars, but some distinctive ones have elicited consensus at least among proponents of the so-called “framework” reading of *On Certainty*, which is, to date, the most accurate one from an exegetical point of view.[[6]](#footnote-6) These criteria are:

1. Hinges play a normative role—let it be a meaning constitutive one (e.g. “Here is a hand”, or “*a* is a physical object”) or the role of norms of evidential significance (e.g. “The Earth has existed for a very long time”, which needs to stay put for us to be able to engage in geological or historical inquiries, for instance).[[7]](#footnote-7) Irrespective of whether hinges are neither true nor false, or true in an entirely “empty” sense,[[8]](#footnote-8) and whether or not they could be considered propositions, even while failing at bipolarity, since, at most, they would be “unipolar”,[[9]](#footnote-9) their function in context is normative, not descriptive. Furthermore, whether or not hinges are effable,[[10]](#footnote-10) they would play a normative role, and, when spoken *qua* hinges, they would be mentioned only to teach or remind subjects of their role within our language and epistemic practices.
2. Hinges cannot be epistemically justified—either because all evidence would presuppose them (like in the case of “The Earth has existed for a very long time” *vis-à-vis* ordinary propositions of geology or history, like “There were dinosaurs on Earth over 230 million years ago”; or “There are physical objects”, *vis-à-vis* the proposition “There are two chairs in this room”); or because they would not be any more certain after checking than they were before. For example, “Here is my hand” or “My name is …” are no less certain than the sensory or mnestic evidence I could respectively use to determine their truth (cf. OC 125). Therefore, they cannot be the object of knowledge. If knowledge is predicated of them, then it is an expression of objective certainty (OC 270-272) and hence the remark “I know” would play a grammatical role (OC 58), rather than an epistemic one. For the sake of perspicuity, it could be safely replaced with “It stands fast for my and anyone else” (OC 116, 125, 144, 151-152, 234-235) or “Here is a mistake is impossible”, or even “I can’t be wrong about that” (OC 15-17, 22, 25-26, 54, 155, 194, 425, 497, 630-631, 633-638, 641-651, 659-660, 662-676).
3. Hinges cannot be doubted, for all our evidence speaks in favor of them and none against them (OC 117-118, 191, 203). Evidence speaks in favor of them, yet, for the reasons briefly mentioned in (2), it cannot prove them or justify them. Nor could evidence speak against them since everything we regard as evidence depends on taking them for granted.
4. We acquire them as part of an image of the world we inherit from the upbringing we receive as members of a community that shares various epistemic practices and a language (OC 93-97, 162, 167, 233, 262). Notice, however, that by itself this criterion is insufficient to single out hinges from non-hinges. There are plenty of things we acquire through our upbringing in our communities, which would not plausibly be regarded as hinges. For instance, which side of the road one should drive, or whether it is permissible to take a right turn, by car, at a red traffic light, after stopping and checking no one is coming from the opposite direction, or whether the weather is mostly sunny or rainy are all things we pick up in this way, yet would be either mere conventions or mere empirical generalizations, respectively. (4) is therefore relevant to demarcate hinges only in connection with (1)-(3).
5. *Mathematical hinges?*

With this characterization in mind, let us consider whether “2x2=4” (or “12x12=144”) could count as a hinge. The discussion will first be carried out while taking for granted Wittgenstein’s conception of mathematical propositions as rules, whose meaning is determined by their proof, with the attendant consequence that false mathematical sentences are in fact meaningless, for, clearly, they couldn’t be proved. We will then consider if there could be mathematical hinges, as opposed to ordinary mathematical propositions, on a more traditional understanding of mathematical propositions as statements about natural numbers (in the case of arithmetic), which are either true or false and whose meaning is not established by their proof.

* 1. *Could there be mathematical hinges if all mathematical propositions are rules?*

Let us therefore start by considering whether “2x2=4” would satisfy (1). Clearly it would, at least in the sense that if I think that there are 2 pears in each package and I know I have bought 2 packages, if, by counting, I reached the result that I have 5 pears in total, while no one has messed with my pears, I will conclude that I have miscounted, and not that it might after all be the case that 2x2=5. Yet, the same would go for “1,057x216=228,312”. Of course, not being such a simple multiplication, I could also double-check it, but once I have double-checked it, if I had counted 228,313 apples, I would conclude that I have made a mistake in counting (assuming that no one has messed with my apples), and not that it might be the case that 1,057x216=228,313. Being rules, however, neither “2x2=4” nor “1,057x216=228,312” would be true; or else, if they were, they would both be true in an entirely “empty” way.

Let us now move to (2). The kind of verification we are concerned with when mathematical propositions are at stake is not an empirical one. We don’t hold that “2x2=4” as the result of an empirical generalization. That is, we don’t hold that because by counting 2 apples in each of 2 boxes several times we have always arrived at 4. In the mathematical case, a verification would have to be a proof (cf. OC 563). However, if that is the case, we may just as well prove that 2x2=4 or that 1,057x216=228,312. Furthermore, the evidence I would have for “2x2=4”—namely a proof—would be as certain as the one I would have for “1,057x216=228,312”. The latter may be a bit more laborious, but it would not be any different, and certainly not any less certain than the former.

To that, one may object that the relevant evidence need not be a formal proof, but could be something like counting. If so, one may maintain that it would not be any more certain that I have counted 4 apples, while counting the apples in my packages, than that 2x2=4. By contrast, counting up to 228,312 apples in the other case would be more certain than doing the multiplication. To which I would reply “It depends”. Typically, young children, who learn multiplications (and multiplication tables) after learning how to count, would go through a phase in which they would feel more certain about the latter than about the former. But that should normally change after a while, once one has become conversant with multiplication tables or with various multiplication techniques. I, for one, would feel much more certain about doing the multiplication, than about counting, for counting would be more subject to the possibility of miscounting (I could count the same apple twice, or repeat the same numeral in the sequence twice, or forget where I have arrived in the sequence and skip one or more numbers, etc.). What is more, these are entirely psychological or subjective senses of “being certain”, and clearly Wittgenstein is not concerned with subjective certainty when hinges are concerned (OC 194, 415, 563).

Considering now whether these alleged mathematical hinges would have to be presupposed for us to have any mathematical evidence—let it be a proof or counting—it seems clear that this is not so. We can count without holding fast to these elementary propositions and a formal proof would not depend on taking them for granted. Of course, if we are considering complex multiplications, having learnt by rote basic ones like “2x2=4” will certainly expedite our calculations, but they wouldn’t be impossible without relying on, or taking for granted those basic ones.[[11]](#footnote-11)

Thus, for this very same reason, alleged mathematical hinges, contrary to “real” hinges, need not be presupposed to resolve mathematical doubts. In that sense, they would fail to comply with criterion (3) for being a hinge. Yet, it remains that, after going through usual acculturation processes, “2x2=4” or “12x12=144” would stand fast for me and for all of us and that nothing we know would seem to speak against them.

Finally, and connectedly, it is true that “2x2=4” is passed on to us as part of our world-picture in the sense that it is an elementary multiplication we have been shown hundreds of times. In that sense, we are certainly more familiar with it than with “1,057x216=228,312”. Hence, it does meet condition (4).[[12]](#footnote-12) However, it is also part of our world-picture that more complex multiplications can be made, utilizing simple techniques we all learn in school, or that they can be made quite easily by using a calculator (even that was not the case at the time when Wittgenstein was writing). True, it is more likely that we test the reliability of a calculator by seeing what it does when we input “2x2”, rather than when we input “1,057x216”. Yet, again, this is simply a function of how familiar we are with that multiplication. That is, there is nothing that prevents us, in principle, from using the latter multiplication to test the reliability of the calculator. Hence, the fact that we use “2x2” rather than “1,057x216” does not point to any substantive difference between these two arithmetical propositions. That is, it cannot be used to mark a difference *in kind* between those multiplications, but only *in degree* of familiarity and complexity. Typically, however, hinges are not just different from ordinary empirical propositions in their degree of familiarity or complexity, but in kind. For they exhibit features (1)-(3) while ordinary empirical propositions, not even the ones we are more familiar with, do not. Yet, in the mathematical case, given Wittgenstein’s own understanding of mathematical propositions and of hinges, we haven’t found a way of defending the idea that, among mathematical propositions, there would be some that would partake of (1)-(3), while other ones wouldn’t. Thus, I submit, we haven’t found a way of characterizing the distinction between mathematical hinges and non-hinges.

To sum up. Given Wittgenstein’s conception of mathematical propositions, “2x2=4” or “12x12=144” are rules, but so is “1,057x216=228,312”. Contrary to real hinges, “2x2=4” or “12x12=144” can be evidentially supported, since they can be proved, but so can be “1,057x216=228,312”. Different kinds of evidential support, like counting or adding, would behave similarly in the two cases, and after a little while also multiplying “1,057x216” to get “228,312” could become “second nature” to us, just as it is second nature to us to hold “2x2=4”. Moreover, these alleged hinges do not seem to be necessarily presupposed by anything we regard as a proof, let it be a formal one or a more informal one based on counting or other techniques. Or, if they are, it is only to make the latter proofs more expedite. Yet, they are not presupposed by any possible resolution of a mathematical doubt. The main difference between “2x2=4” or “12x12=144”, on the one hand, and propositions like“1,057x216=228,312”, on the other, seems to reside only in the fact that we are more familiar with the former than with the latter, as a result of our being acculturated within a form of life that passes them onto us from very early on.[[13]](#footnote-13) Hence, while familiarity is certainly a characteristic trait of real hinges too, it is not enough to turn propositions like “2x2=4” or “12x12=144” into mathematical hinges.

* 1. *Could there be mathematical hinges if mathematical propositions aren’t rules?*

Let us now consider a more traditional conception of mathematics,[[14]](#footnote-14) according to which mathematical propositions are statements about natural numbers (in the case of arithmetic), which are either true or false and whose meaning is not established by their proof, to see whether it could help us draw a distinction between mathematical hinges and non-hinges.

Mathematical hinges should stand out because, contrary to other mathematical statements, they would be rules, rather than descriptions of mathematical states of affairs; hence, they alone would be neither true nor false, or true only in an “empty” sense of the term (1), while “ordinary” mathematical statements would not be true in an entirely “empty” sense of the term (when in fact true). Connectedly, mathematical hinges could not be evidentially supported—that is, proved—while other mathematical statements would be provable (2) and, when proved, they would count as true in a “substantive” sense of the term, albeit in an evidentially constrained one rather than in a correspondentist fashion. Moreover, they would have to be presupposed by anything we regard as evidence in mathematics and, for that reason, they could not be doubted, for any reason to doubt them would presuppose them (3). Finally, they alone would have to be part of a world picture we have inherited by being trained within our community (4).

While we can certainly conceive of such a contrast between mathematical hinges and non-hinges, it is quite clear that “2x2=4” or “12x12=144” would not be good examples of the relevant category. For why should they be rules, rather than descriptions of mathematical states of affairs, such that they should be considered neither true nor false, or true only an “empty” sense of the term? After all, they too would be provable and would not be themselves presupposed by any proof of them. Hence, they would be knowable and dubitable just like “1,057x216=228,312”. Of course, we are more familiar with them, as a result of our inculturation within a community that passes them onto to us since very early on, often asking us to learn them by rote, but, as before, this is not enough to turn them into categorially distinct mathematical statements.

Thus, I submit, on a Wittgensteinian account of mathematics, the very distinction between mathematical hinges and non-hinges can hardly be drawn. On a more traditional understanding of mathematics, in contrast, that distinction can be drawn conceptually, but then it cannot be exemplified by appealing to the mathematical statements that Wittgenstein mentions in *On Certainty* and that should allegedly count as examples of mathematical hinges. For, apart from (4), they do not meet any of the criteria that single out hinges from non-hinges for Wittgenstein. In §6, I will propose to exemplify that distinction differently. But, before turning to that task, let me say a bit more on the actual role of mathematical statements like “2x2=4” and “12x12=144” in *On Certainty*.

1. *Objects of comparison only*

Contrary to what several interpreters have maintained,[[15]](#footnote-15) then, I don’t think we can consider Wittgenstein’s mathematical examples in *On Certainty* as instances of mathematical hinges.[[16]](#footnote-16) That raises the question of why he considered them, or of what role they played within the context of that work (however fragmented it is). The suggestion I would like to explore is that they have an elucidatory function. By comparing hinges to them, Wittgenstein aims to bring out important features of hinges while not going as far as holding that there would be cases of mathematical hinges.

Indeed, in one of the remarks in which mathematical examples are introduced, Wittgenstein invites us to make a comparison. He writes (OC 447):

Compare with this [“This is my hand”] 12x12=144. Here too we don’t say “perhaps”. For, in so far as this proposition rests on our not miscounting or miscalculating and on our senses not deceiving us as we calculate, both propositions, the arithmetical and the physical one, are on the same level. I want to say: the physical game is just as certain as the arithmetical.

Notice that here Wittgenstein is not taking “12x12=144” for granted. Rather, he is introducing it as a simple arithmetical equation that we may calculate, presumably by writing it down; or that we may resolve by counting. These processes, however, would themselves rely on the fact that our senses, while we carry them out, do not deceive us. Thus, the arithmetical proposition, which is so certain for us, is seen as the result of a process that presupposes the existence and the stability of physical objects, let them be signs on paper, or apples or pears in boxes we may be counting, as well as the reliability of memory as we go along (cf. OC 337).

Furthermore, Wittgenstein himself warns us that the comparison could lead to some misunderstanding, for he continues (OC 447) “But this can be misunderstood. My remark is a logical and not a psychological one” (cf. OC 43).[[17]](#footnote-17) I take it that, with this *caveat*, Wittgenstein is pointing out that the certainty we display with respect to “This is my hand” is not, ultimately, of a psychological kind. That is, it is not because we are very familiar with that proposition, or indeed with “12x12=144”, that “the physical game is just as certain as the arithmetical one”. Rather, it is due to the role that that proposition plays with respect to language and further human activities. (I will presently return to this issue).

Thus, the point of the analogy between hinges like “This is my hand” and “12x12=144” is to familiarize the reader with the idea that, contrary to a typically Cartesian picture of certainty, propositions about our bodies and our physical surroundings can, at least in context, be as certain as arithmetical statements. In other words, the purpose of the analogy is clearly “therapeutic”. For Wittgenstein aims to dislodge the idea that propositions about material objects are always open to doubt, as opposed to propositions of arithmetic, or to traditional philosophical truths, such as Descartes’ *cogito*, or indeed to propositions about sense data (OC 90, 426), that traditional empiricists would put at the foundation of knowledge. By so doing, he is going against all traditional foundationalist projects in epistemology.[[18]](#footnote-18) In this particular series of passages, he is achieving his aim by stressing the similarities between propositions about physical objects and arithmetical ones, to bring out the “strength” of the former, as it were, at least in certain contexts. He does so by showing, at once, how empirical propositions can be as certain as arithmetical ones, and how the latter are themselves resting on human activities that depend on the existence and stability of physical objects, and that could, in principle, be affected by errors and mistakes, like the former.

Wittgenstein reiterates and reinforces the idea in the following passage (OC 448):

I want to say: If one doesn’t marvel at the fact that the propositions of arithmetic (e.g. the multiplication tables) are ‘absolutely certain’, then why should one be astonished that the proposition “This is my hand” is so equally?

For, to repeat, the arithmetical propositions too are obtained through processes, like calculation and counting, which presuppose the existence and stability of physical objects, as he stressed in (OC 447), and which are themselves subject to error. Yet, we do not seem to be preoccupied by this fact in the mathematical case. So why should we be worried about this possibility in the case of “This is my hand” (at least in context)? Thus, Wittgenstein is indirectly introducing the idea, he develops in subsequent passages as we shall see, that just because a doubt about the existence of my hand is logically (or metaphysically) possible, it is not any less certain that, at least in Moore-like contexts, there is indeed my hand here.

He then modulates the original idea by bringing in the context of teaching and learning, by writing (OC 449): “Something must be taught as a foundation” (cf. OC 455: “We learn with the same *inexorability* that this is a chair as that 2x2=4”), which is important to stress another analogy between the physical and the arithmetical game. Namely that certain judgements—either about physical objects or about arithmetic—are passed on to us from very early on and that we “swallow” (OC 143) them both without questioning either of them. Why is this important?

Wittgenstein insists on two key ideas. First, that certain judgements are taught as paradigmatic cases of correct applications of terms or of calculus, such that, were they wrong, we would no longer know the meaning of the words we are using (OC 61, 114-115, 126-129, 156, 306, 329, 369-370, 492-496, 519, 522), or what multiplying and calculating could be (OC 38-39, 44-50, 55, 212, 217, 303). Secondly, he points out that we use certain objects to ostensively define the meaning of our terms (OC 450). For this reason, “a doubt that doubted everything would not be a doubt” (OC 450), for the words figuring in it would just have an appearance of meaning. That is, if I seriously doubted, in a Moore-like context, that this is my hand, it would become unclear what I meant by “hand”. Therefore, my doubt—“Is this really a hand?”—would only have an appearance of meaning, but would in fact be meaningless.

Thus, by OC 450, the therapeutic aim of the analogy has been fully achieved by showing that, in Moore-like contexts, “This is my hand” is just as certain as “12x12=144”. For, to doubt of it would deprive words of their meaning, thus making that doubt nonsense, just as doubting that 12x12=144 would deprive us of our multiplication techniques by means of which any doubt about mathematical statements of the same sort could be resolved.

The impossibility of a mistake and the understanding of where such an impossibility comes from are then the main themes in a series of passages starting at OC 651. Writes Wittgenstein:

I cannot be making a mistake about 12x12=144. And now one *cannot* (my italics) contrast mathematical certainty with the relative uncertainty of empirical propositions. For the mathematical proposition has been obtained by a series of actions that are in no way different from the actions of the rest of our lives, and are in the same degree liable to forgetfulness, oversight and illusion.

Both arithmetical statements and hinges like “This is my hand” are grounded in human activities, but they are removed from doubt and inquiry. Since this is clearer in the arithmetical case, likening “This is my hand” to “12x12=144” helps us see the revolutionary point about the certainty of the former Wittgenstein is making and that he condenses in OC (653), where he writes: “If the proposition 12x12=144 is exempt from doubt, then too must non-mathematical propositions be”.

And he immediately considers possible objections, which would maintain the more traditional view according to which “12x12=144” would be more certain than “Here is a hand” because “it is a mathematical proposition” (OC 654). Yet, this fact, by itself, is irrelevant for Wittgenstein, who insists on the conventional elements of mathematical activity, by stressing that the inexorability of mathematical statements is the result of a human decision. He writes: “the mathematical proposition has, as it were officially, been given the stamp of incontestability” (OC 655), and he does use the metaphor of hinges to bring out or stress this point. For he continues “Dispute about other things; this is immovable—it is a hinge on which our dispute can turn”. Yet, I claim, the use of the hinge metaphor should not delude us into thinking that simple arithmetical statements are in fact hinges. Rather, it is meant to bring out the analogy between hinges and mathematical statements by stressing that the incontestability of mathematical statements is not of superior a status than that of “This is my hand” (in context). Just as he stressed this very same point before, by reminding us of the fact that the mathematical statement is the result of a calculation which presupposes the existence and stability of physical objects and is in principle open to error and illusion just as seeing (or seeming to see) a hand, he is now making the same point by stressing the element of decision, and, in a sense, of arbitrariness, that makes mathematical statements exempt from doubt. That is, we could go on asking and inquiring whether it is really the case that 12x12=144, or that “This is my hand” in Moore-like situations. But even if this is possible in principle, we do not do it. That we do not do it is clear in the mathematical case. Once we bring that in view and we start seeing “This is my hand” in this new perspective as playing a role similar to the one of mathematical statements, it becomes clear in the physical case too (or indeed in the case of other hinges Wittgenstein is presenting in *On Certainty*). That this is the point of the analogy it is quite clear in OC 657 (cf. also OC 303-306), where he writes:

The propositions of mathematics might be said to be *fossilized*. The proposition “I am called…” is not. But it too is regarded as incontrovertible by those who, like myself, have overwhelming evidence for it. And this is not out of thoughtlessness. For, the evidence’s being overwhelming consists precisely in the fact that we do not *need* to give way before any contrary evidence. And so we have here a buttress similar to the one that makes the propositions of mathematics incontrovertible.

Notice that here Wittgenstein is not talking about a subset of mathematical propositions—that is, alleged mathematical hinges. He is talking about mathematical propositions in general, which we regard as certain even in the face of contrary evidence, like in the case of counting 5 apples or, indeed, 228,313 apples, after multiplying 2x2 or 1,057x216. And just as these mathematical propositions are fossilized, which means that we would rather think that either we have miscounted or that someone has messed up with our apples, so “My name is …” is practically fossilized for each of us, at least most of the times. That is, we would normally recuse any evidence that we might have been mistaken about our own name. For that would entail that everything we have been told by family members, or that we might have seen written in official documents, would be wrong. Now, such a terrible prank could have been pulled on us, but it is extremely unlikely. That is why we are most of the times entitled to treat “My name is …” on par with “2x2=4” or “1,057x216=228,312”. Yet, a difference remains between the two cases for “My name is …” is mostly, or normally, or practically fossilized for each of us, while those mathematical statements are officially or always so fossilized.[[19]](#footnote-19) That is why we might think of them as being necessarily true. But, for Wittgenstein, their superior strength would only be a function of a more definitive decision, as it were, and would not be grounded in anything like the metaphysical necessity of mathematical statements. As he writes, “doubt gradually loses its sense” (OC 56). “My name is …” and “2x2=4” or indeed “1,057x216=228,312” are very close on this continuum, for Wittgenstein, contrary to what traditional epistemologists or indeed philosophers of mathematics might have held. Yet, they are still somewhat different. Yet, such a difference, for Wittgenstein, does not in any way justify or leave the door open to skeptical assaults. The rest of *On Certainty*, till the final remark in OC 676 about the very intelligibility of the dreaming hypothesis, is intended as dismantling the possibility—better: the intelligibility—of a radically skeptical doubt about propositions like “Here is my hand” or “My name is …”.

Thus, to sum up. Far from proposing the idea that there could be mathematical hinges, as opposed to mathematical non-hinges, or even that mathematical propositions and propositions about physical objects, or our own names, and many others are absolutely identical at least in context, Wittgenstein is just trying to dislodge the idea that they are completely different, such that a radical doubt about the latter could be intelligible, after all.

It is perhaps worth connecting this finding with a reminder about Wittgenstein’s peculiar methodology.[[20]](#footnote-20) Namely, a methodology that, by illuminating similarities and differences between apparently distant objects, aims, at once, to achieve a perspicuous (re)presentation (*übersichtiliche Darstellung*) of them, while dismantling long-lasting philosophical mis-representations of them. If Wittgenstein is right, then, we do have certainty outside the mathematical realm (as well as outside the realm of pure ratiocination or of our inner world), this certainty is a function of the normative role certain propositions play, both in the sense of meaning-constitutive norms and of norms of evidential significance, at least in context, which makes them impervious to skeptical assaults. To liken them to mathematical propositions, that wear these features on their sleeves, as it were,[[21]](#footnote-21) helps us represent them correctly and see more clearly why they would be unassailable. For skeptical doubts would ultimately be irrational—that is, unsupported by evidence and reasons—and even meaningless, since reasons for them and even the possibility of stating them would presuppose those hinges they attempt to attack. That is why, for Wittgenstein, skeptical doubts are ultimately delusional, or, equivalently, merely illusions of doubt.

1. *Mathematics as a hinge?*

One may suggest that Wittgenstein thought of mathematics as a whole as a hinge.[[22]](#footnote-22) There are two ways of interpreting this suggestion. On the one hand, one may think that, for Wittgenstein, all mathematical propositions would be hinges. On the other, one may hold that mathematics as a discipline or a practice was a hinge for him. On reflection, however, it is difficult to make sense of this suggestion either way. For, on the former reading, it may be conceded that all mathematical, in fact, arithmetical propositions, would be rules of meaning and/or evidential significance; but not all of them would be indubitable and unjustifiable. False or complex mathematical statements would be dubitable and all mathematical statements would admit of a proof. Furthermore, we do not, as we could not acquire the entirety of mathematical statements as part of an inherited world-picture.

One might then suggest that all and only mathematical theorems – that is, proved mathematical propositions – would be hinges for Wittgenstein, for their proof would determine the sense of the theorem – that is to say, its meaning.[[23]](#footnote-23) This is certainly a subtle issue, pertaining to the overall interpretation of Wittgenstein’s philosophy of mathematics, which exceeds the scope of this paper. Notice, however, how distant this idea would be from the original one, typically attributed to Wittgenstein – namely, that only some mathematical statements, by virtue of their being acquired as part of a shared picture of the world, which is passed on to us through inculturation, would play a hinge role. Furthermore, the idea that we would acquire all theorems, as part of a shared and inherited world-picture, is moot, whether it could be reconciled with Wittgenstein’s position and examples in *On Certainty* or not.

On the latter reading, prospects look better. For mathematics as a discipline, with various calculation techniques, would be acquired through our upbringing within a community. It would also be part of the world-picture inherited in that way that mathematics provides us with rules of evidential significance. Mathematics as a practice or a discipline would not be justifiable and it would itself be used to provide justifications for some specific beliefs (for example, that we must have made a mistake or that someone must have messed with our grocery if, after buying 2 packs of pears with 2 pears each, we counted only 3 pears, once back home). Nor could mathematics as a discipline be doubted, for nothing we do or know speaks against it. Yet, there is no hint in *On Certainty* that practices or entire disciplines would be hinges. Rather, the idea is always that specific propositions play a hinge role, vis-à-vis ordinary propositions within a given domain (let it be the empirical domain or otherwise), because they are constitutive of a given epistemic practice (OC 167). Hence, it seems doubtful that this suggestion would be useful to understand the idea of mathematical hinges in *On Certainty*. Moreover, it is not particularly tenable as such, because it would over-generalize (considerations similar to the ones just outlined could be advanced for science, and, at least relative to some people or cultures, also for religion, etc.). Furthermore, it would not tell apart propositions within mathematics (or science or religion) that need to be assumed for the whole discipline and practice to be possible (or to make any sense at all) and those propositions which, within mathematics (or science or religion), are open to doubt, verification and control.

Thus, though ingenious, I actually don’t think that this suggestion could be made to work, either as an exegesis of Wittgenstein or as a viable “hinge” philosophy of mathematics. Let us therefore explore another avenue.

1. *Axioms as hinges?*

If we go back to §3.1 and the kind of contrast we introduced there between ordinary mathematical propositions and hinges, on a rather traditional understanding of mathematics, it is not difficult to see how it could be instantiated. For, clearly, a mathematical theory’s axioms could be taken to fulfill the role of hinges.

To be more precise, let us look at the contrast once more. We said that mathematical hinges should stand out because, contrary to other mathematical statements, they would be rules, rather than descriptions of mathematical states of affairs; hence, they alone would be neither true nor false, or would be true only in an “empty” sense of the term (1). Connectedly, they alone could not be evidentially supported—that is, proved—while other mathematical statements would be provable (2) (and, when proved, true in a more robust sense of the term). Notice that this would still be the case if, within a given theory, some of the propositions that we normally derive as theorems were in fact used as axioms to derive the propositions we typically use as axioms. Thus, the point would be that being a hinge, within a mathematical theory, does not have to do with the specific content of the mathematical proposition under consideration, but with the role a given proposition plays (or would play) within the theory. Moreover, axioms would have to be presupposed by anything we regard as evidence within a given mathematical theory and, for that reason, they could not be doubted, for any reason to doubt them would presuppose them (3). Finally, they alone would have to be part of a world picture we have inherited by being trained within our community (4).

Now consider the role of axioms in, for instance, Euclidean geometry. Clearly, they play a meaning constitutive role. For they tell us how, for instance, a point or a line behave (with respect to other objects in the theory) and therefore, under a functional conception of meaning, what the meaning of these terms is.[[24]](#footnote-24) Furthermore, they cannot be proved within the theory, while any proof proceeds from them, or must take them for granted. Hence, they are also rules of evidential significance, for they establish how something may or may not be proved within a theory. For these reasons—one might hold—either they are neither true nor false; or else, they are true only in an entirely “empty” sense of the term. Connectedly, one might also hold that, within a given theory, they are indubitable because anything we regard as evidence within the theory would presuppose them and therefore nothing within the theory could speak against them, while everything speaks for them. Finally, they might not be as commonly known as basic additions or multiplications are, yet these theories are passed on to us through inculturation within a community that assigns an important role to mathematics, understood as a highly specific scientific discipline, whose rudiments, such as Euclidean geometry, are imparted in school. The canonical presentation of these theories would then single out those propositions which, playing the role of axioms within them, would be transmitted as hinges to us. Thus, while axioms are not like common-sensical truisms, they are comparable to other scientific hinges we find in *On Certainty*, like “Water boils at 100 ˚C”, which we absorb in school as elements of scientific theories we are exposed to. Furthermore, it may well be the case that, at least with respect to geometry, those propositions we typically consider as the theory’s axioms may have been fossilized from experience. In that sense, it may well be the case that our choice of axioms, while revisable in principle, for, as we noticed, any theorem can be turned into an axiom and vice versa, may nevertheless appear more natural to us than its alternatives. Whatever their genealogy might be, it remains that the propositions elected to the role of axioms would play a rule-like role. Furthermore, they would serve as the starting points from which all others theorems within the theory would be derived. Finally, they could not be doubted, within the theory, unless an inconsistency resulted from them, for any theorem within the theory would derive from them and would therefore presuppose them.

Of course, as we know, they may be doubted from outside the theory, as the vexed issue of Euclid’s parallel axioms shows. But that would resemble in many ways the scenarios Wittgenstein repeatedly envisages in *On Certainty* in which we encounter people that endorse different hinges (OC 106-108, 92, 132, 239, 286, 336, 608-612, 667, 671). While a treatment of that kind of hinge disagreement falls outside the scope of this paper,[[25]](#footnote-25) the important point is that we have found a way not only of making sense of the distinction between hinge and non-hinge mathematical propositions (or statements) but also of exemplifying it.

Whether this is a viable conception of the relationship between mathematical axioms and non-axioms is not the present issue. To remain within Wittgenstein scholarship, however, it is clear that the present account would have important repercussions on the assessment of Wittgenstein’s philosophy of mathematics. For, if what I have been saying is along the right lines, then not all mathematical statements would be rules. Only axioms would be, and they would play a hinge-like role with respect to other mathematical statements. The latter, contrary to the *vulgata* regarding Wittgenstein’s conception of mathematics, would be true or false, depending on whether there is a proof of them (or of some other mathematical statement incompatible with them). Hence, contrary to what Marie McGinn stated (1989: 128), they would not be true in an entirely “empty” sense of the term, but in quite a robust—though evidentially constrained—one. Furthermore, only axioms would be indubitable, at least if one remains within a given mathematical theory, and as long as no inconsistency resulted from them, for no evidence within the theory could speak against them and everything, which is regarded as evidence within the theory, would speak for them. Moreover, while ordinary mathematical propositions that are proved would be meaningful and true, albeit in an evidentially constrained way, unproved mathematical propositions could still be meaningful, and such as to motivate doubts and inquiries with respect to them.

Another significant point of departure would be the fact that a hinge philosophy of mathematics like the one just sketched would apply to axiomatized theories, and in fact to canonical presentations of them. By contrast, Wittgenstein, in the *Remarks on the Foundations of Mathematics*,[[26]](#footnote-26) seemed to be more interested either in simple mathematical equations, or in criticizing set theory and certain notions of infinity. The “anthropological” aspect of his philosophy of mathematics that many interpreters have stressed, which would have to do with the applicability of mathematical statements, would no doubt be lost on the present account, at least in part. Yet, as noticed, at least some axiomatized theories are routinely imparted in schools and are passed on to us through the same kind of upbringing from which we inherit our world-picture. The latter, for Wittgenstein, does contain hinges like “Water boils at 100 °C” (OC 293, 567, 599), geographical and historical notions and other truths we get from textbooks (OC 159, 162-163, 185, 234, 599-600). Furthermore, as we also remarked, at least in some cases, like Euclidean geometry, the applicability of the theory to real-life problems could have motivated not only inquiry but also axioms’ choice.

Whether or not this conception of mathematical hinges, as opposed to non-hinges, could constitute the backbone of a viable philosophy of mathematics or of a philosophy of mathematics that would have encountered Wittgenstein’s favors, it can confidently be maintained that it is—in broad strokes—what a possible “hinge philosophy of mathematics” would look like.

**References**

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Kusch, M. 2016 “Wittgenstein on mathematical certainties”, *International Journal for the Study of Skepticism*, 6, 120-142.

McGinn, M. 1989 *Sense and Certainty. A Dissolution of Scepticism*, Oxford, Blackwell.

Moyal-Sharrock, D. 2005 *Understanding Wittgenstein’s* On Certainty, London, Palgrave.

Williams, M. 2004 “Wittgenstein, truth and certainty”, in M. Kölbel and B. Weiss (eds.) *Wittgenstein’s Lasting Significance*, London-New York, Routledge, 249-284.

Williams, M. 2005 “Why Wittgenstein isn’t a foundationalist”, in D. Moyal-Sharrock and W. H. Brenner (eds.) *Readings of Wittgenstein’s* On Certainty, London, Palgrave, 47-58.

Wittgenstein, L. 1969 *Certainty*, Oxford, Blackwell.

Wittgenstein, L. 1978 *Remarks on the Foundations of Mathematics*, Oxford, Blackwell.

Wright, C. 1985 “Facts and certainty”, *Proceedings of the British Academy*, 71, 429-472.

1. \* I would like to thank … for helpful comments on previous versions of this paper, as well as .. and other people in attendance at the conference … for stimulating discussion that helped me improve the paper. [↑](#footnote-ref-1)
2. I think it is interesting, and important, that he is not choosing basic additions like “1+1=2” or “2+2=4”, or the Kantian “5+7=12”. For Wittgenstein seems to be thinking of multiplications, typically learnt in school, often by means of multiplication tables and memorized, rather than of simple additions that may not depend on such acculturation processes. This would stress an element of commonality between some hinges, like “Water boils at 100 °C”, or “Napoleon won at Austerlitz”, and these mathematical statements, which would be likewise “swallowed down” (OC 143) in the process of acculturation. There is, however, a problem of individuation of what would count as a mathematical hinge, if there were any (cf. also fn. 10). For expository reasons, I will be sticking to Wittgenstein’s own examples, which I will take as possible instances of mathematical hinges. I will then assess whether they could really play that role, once the criteria for being a hinge are clearly laid down. [↑](#footnote-ref-2)
3. Of course, this is a very crude rendition of Wittgenstein’s position, but it captures the essence of his distinctive view of mathematics. It is true that for him it was important for mathematical propositions to be applicable. Thus, his wasn’t a crude form of conventionalism. It remains, however, that he insisted on the normative—as opposed to descriptive—role of mathematical propositions (or statements). [↑](#footnote-ref-3)
4. In the following, I will use the expressions “mathematical propositions” and “mathematical statements” largely interchangeably. Whenever relevant, I will differentiate between the two, and explain why, on some interpretations of Wittgenstein’s later philosophy, these expressions would not be equivalent. [↑](#footnote-ref-4)
5. Most notably McGinn (1989) and Moyal-Sharrock (2005). But also, more recently, Kusch (2016). [↑](#footnote-ref-5)
6. Wright (1985), McGinn (1989), Moyal-Sharrock (2005) and … are prominent versions of the framework reading. [↑](#footnote-ref-6)
7. Kusch (2016) criticizes the framework reading by insisting on the fact that mathematical hinges would not be meaning constitutive and would be provable. The former point is actually quite irrelevant since it has been amply shown (since Wright 1985) that hinges would play a normative role with respect to how to interpret evidence, and not merely in a semantic sense. The latter point, in contrast, would speak against framework readings only if there were mathematical hinges, which is the very thesis the present paper is calling into question. [↑](#footnote-ref-7)
8. Moyal-Sharrock (2005) and …. endorsed the former reading. McGinn (1989), Williams (2004) and (more tentatively) …. put forth the latter, more liberal one. [↑](#footnote-ref-8)
9. Moyal-Sharrock (2005) is the staunch supporter of this more austere reading according to which if something is a rule, it is not a proposition. …, in contrast, opts for the more liberal one by stressing the fact that, since the *Philosophical Investigations*, the notion of a proposition, for Wittgenstein, is a family resemblance one and that in several passages in *On Certainty* he insists on the very concept of a proposition is not “a sharp one” (OC 320). [↑](#footnote-ref-9)
10. Notoriously, Moyal-Sharrock (2005) denies this and holds that hinges, *qua* hinges, are manifested only in action. … dissents. While all our language games are grounded in action (and reactions), for Wittgenstein, it would be hard to maintain that alleged mathematical hinges, like “2x2=4”, would be manifested mostly in action. Rather, they would normally be employed in calculations and thus in entirely ratiocinative processes, which are often verbalized. Thus, making ineffability a distinctive criterion of “hinginess” would make it quite trivially the case that Wittgenstein’s mathematical examples would not count as instances of mathematical hinges. Kusch (2016) seizes on this criterion to utilize Wittgenstein’s mathematical examples against framework readings. Again, this would follow only if these mathematical statements counted as hinges, which is what is presently under dispute. [↑](#footnote-ref-10)
11. If we used the Japanese method for multiplying, there would be no need to presuppose knowledge of “2x2=4” or the like. See <https://www.youtube.com/watch?v=P7vjtS7PNBc> for a demonstration of how the method works. [↑](#footnote-ref-11)
12. It is more doubtful that “12x12=144” is a hinge. For one thing, it is apprehended later in school and schooling may not be so widespread as to make it a hinge for most of us. For another, “12x12=144” is typically learnt together with similar multiplications, but how far do they go? That is, is “16x16=256” as nearly common knowledge as “12x12=144”, or indeed as “2x2=4”? If it is not, as I surmise, and if there is ample room for personal variability regarding what would count as a mathematical hinge, it becomes even more doubtful that there are any. For it is not even clear which mathematical propositions would figure in the kind of world-picture hinges constitute and that is passed on to us from the rest of our community. Of course, one might try to defend the idea by appealing to the fact that Wittgenstein countenanced also personal hinges, like “My name is …”. But notice that while the dots would be filled out differently for each of us, the form of the proposition would be the same and could lead to a generalized hinge like “Every adult, normal human being knows their name”. This wouldn’t be the case with personal mathematical hinges, for we couldn’t clearly fill in “Every adult, normal human being knows that …”, where the dots should be replaced by a mathematical proposition which is common knowledge. Or else, it would make for a very idiosyncratic distinction between mathematical hinges and non-hinges, like “For me, … is a (mathematical) hinge”, where, by changing the subject the mathematical proposition to be plugged in to replace the dots would change as well. In sum, it is not clear which mathematical propositions would count as mathematical hinges, really. If their number were too small or their content too idiosyncratic, then their philosophical interest and relevance would vanish. [↑](#footnote-ref-12)
13. It may be suggested that the difference resides in the fact that the former, contrary to the latter, are “surveyable”. We “see immediately” why “2x2=4”, while we do not see immediately, if we ever see it at all, that “1,057x216=228,312”. But, again, this would be an entirely psychological fact, assuming it is a fact. For it is noteworthy that children take a while to “see” this allegedly obvious fact. Moreover, it is not clear that we also see immediately why “12x12=144”. If surveyability were what tells mathematical hinges apart from non-hinges, the number of mathematical hinges would shrink considerably and not even “12x12=144”, which is one of Wittgenstein’s favorite examples in *On Certainty*, would probably count as a hinge. [↑](#footnote-ref-13)
14. And I leave it open how close to Wittgenstein’s “real” philosophy of mathematics this conception might be. [↑](#footnote-ref-14)
15. As we have seen McGinn (1989) and Moyal-Sharrock (2005), who defend a framework reading of hinges, think that also some mathematical examples could fall within that category. Kusch (2016) is also in favor of seeing them as hinges, but he then uses these mathematical examples to defend an epistemic reading of hinges in general. [↑](#footnote-ref-15)
16. The only passages that, to my mind, could be used to maintain that reading are OC 48-49 (and OC 655, which I will presently discuss in the text), where Wittgenstein writes “Out of a host of calculations certain ones might be designated as reliable once for all, others as not yet fixed. … Even when the calculation is something fixed for me, this is only a decision for a practical purpose”. However, in OC 50 it becomes clear that the calculations which would be so designated are the ones which have been checked. Thus, this could be equally true of “12x12=144” as well as of “1,057x216=228,312”. [↑](#footnote-ref-16)
17. “Logical” in *On Certainty* is almost always used in the sense of “grammatical”, in Wittgenstein’s sense of the term. [↑](#footnote-ref-17)
18. This is independent of whether Wittgenstein ultimately wanted to replace old foundational projects with a new one, as Moyal-Sharrock (2005) maintains, or not, as Williams (2005) and … have maintained. [↑](#footnote-ref-18)
19. “Fossilized” in this context alludes to the idea that mathematical statements, for Wittgenstein, have been given the stamp of incontestability once and for all (at least when checked). See also fn. 13. [↑](#footnote-ref-19)
20. See …. for a fuller treatment of this issue. [↑](#footnote-ref-20)
21. Or at least on a certain conception of them. [↑](#footnote-ref-21)
22. This is a suggestion that … put forth after reading an earlier draft of this paper. [↑](#footnote-ref-22)
23. I would like to thank an anonymous referee for raising this important point. [↑](#footnote-ref-23)
24. Since Hilbert's work on the foundations of geometry (1899), it is established that the axioms do not univocally determine the meaning of the non-logical constants occurring in them. While the axioms do not tell us what a point or a line are as such, they tell us what plays that role within the theory. I would like to thank an anonymous referee for suggesting the need of such an important qualification. [↑](#footnote-ref-24)
25. I have addressed it in … and, more recently, in …. Broadly speaking, we could think of it either as a disagreement between different theories with their respective hinges; or as a disagreement within one single and evolving theory, in which case the axiom would actually not be considered as a hinge anymore and would be open to revision or abandonment. Similar considerations could be made in the case of the hinge “Nobody has ever been on the Moon” that Wittgenstein discusses in *On Certainty*. [↑](#footnote-ref-25)
26. I am not considering *On Certainty* because, as I have maintained in this paper, I don’t think there is even a sketch of a “hinge philosophy of mathematics” there. [↑](#footnote-ref-26)